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Performance evaluation of an optimized two-stage, free-displacer plastic Stirling cryocooler with gap regenerator

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A computer model developed recently to determine the performance of free-displacer Stirling cryocoolers was extended to include an optimization routine. This was applied to a two-stage, free-displacer Stirling cryocooler to determine the optimized dimensions for the displacer. The design parameters taken for the optimization were the diameters and stroke of the 1st and 2nd displacer. Based on the results of the optimization, a two-stage displacer was fabricated. The performance of this machine was evaluated under different working conditions. The agreement between the predicted and experimental results was found to be very close.

Keywords: Stirling cryocooler; model; optimization

When using a computer simulation of cryocooler performance, the development cost for a specific cryocooler can be significantly reduced. The models for the computer simulation of Stirling cycle machines, developed by Martini[1] and Urieli[2], can be regarded as the starting work in this field. Martini's model was a second order model while Urieli's model was third order, indicating the complexity of the model. Both these models, however, were developed for a single-stage machine. Recently, a second order computer model for a multistage Stirling cryocooler was developed[3]. The model was validated by experimental results obtained from a three-stage Stirling cryocooler. It was then applied to the design of a two-stage Stirling cryocooler after extending its capabilities to include an optimization routine. The design of the two-stage cryocooler is optimized based on certain simplifying assumptions, in order to get a maximum coefficient of the performance at 77 K. The importance of the computer model can be understood from this exercise and one can carry out a parametric analysis of different design and operating parameters before the actual fabrication of the machine is carried out. Based on the results of the optimization, a two-stage cryocooler was built which gave a 1 W refrigeration effect at 85 K. The lowest temperature achieved was 45 K for a 6 bar charging pressure when the frequency of the cryocooler is 2.1 Hz.

Model development and optimization of design

Atrey *et al.*[3] presented a computer model for a three-stage, split-type, free-displacer Stirling cryocooler. The predictions of the model were validated by experimental data obtained from this machine. The computer model was then modified for a two-stage cryocooler. In order to develop an optimum design for the two-stage cryocooler, an optimization routine is incorporated in the model. The model, although quite computer time intensive, is a unique tool for the optimization purpose. Based on the requirements of the design, operating and design parameters, along with optimization criteria, can be varied. This makes the model rather flexible. Atrey *et al.*[4] have earlier presented the results of optimization for a two-stage cryocooler in graphical form.

Figure 1 gives the optimization routine in the form of a flow chart. The data regarding compressor volume, frequency, charging pressure and phase angle between pressure and displacer motion are externally supplied. The optimization routine is mainly designed to determine an optimum combination of the diameter of the 1st stage regenerator and expansion space, $D(1)$, the diameter of the 2nd stage regenerator and expansion space, $D(2)$, and the stroke of the displacer, S_D. The design is based on the optimization of the coefficient of performance of the machine. The program starts with assumed values of $D(1)$ and S_D, and, in an iterative manner, it determines the corresponding value of $D(2)$, according to the criterion of optimization, which also ensures that the net refrigeration effect at the 1st expansion space at 160 K is around 100 mW. Consequently, $D(1)$ and then S_D are varied and the program is executed until the COP of the system is maximum. In the flow chart OP_D and OP_{ST} denote the COP values when diameters are optimized, and when diameters and stroke are both optimized, respectively. The values of $D(1)$, $D(2)$ and S_D for the maximum value of OP_{ST} represent the optimum combination of design parameters.

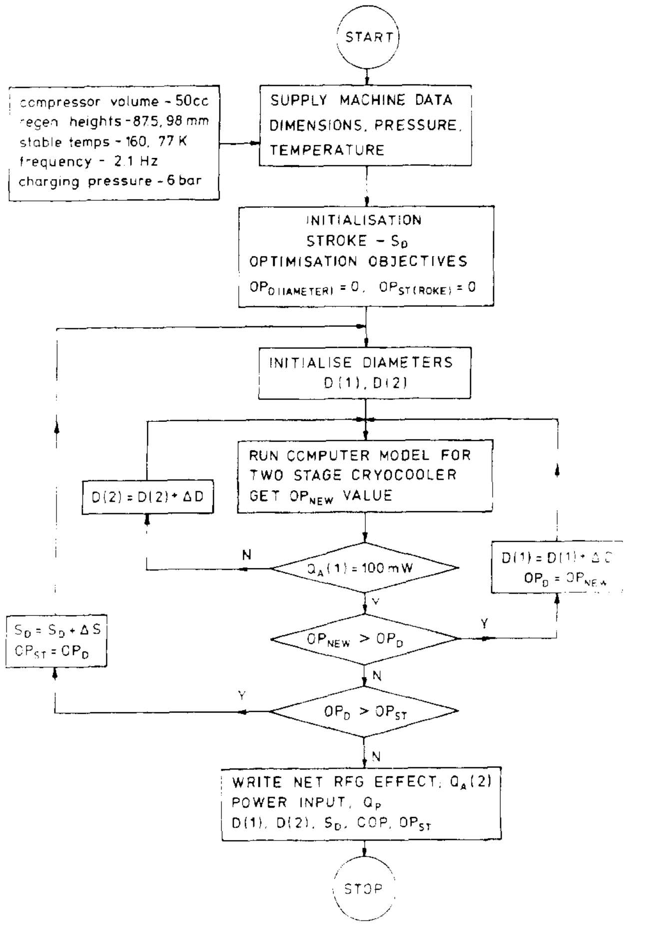

Figure 1 Flow chart for optimization routine

Based on these results of optimisation, an optimised two stage Stirling cryocooler was developed.

Two-stage Stirling cryocooler

Figure 2 shows the two-stage cryocooler. The compressor and the nylon displacer are connected by a connecting tube. The compressed gas, during its travel from the compressor to the expansion spaces, passes through the regenerators which are the surfaces of the displacer, thus using gap regeneration. The displacer is machined to fit, with close tolerance, in a surrounding cylinder of glass-fibre-reinforced plastic of 2 mm wall thickness. To enlarge the effective surface for regeneration, axial slots are sawed into the displacer. The mass of the displacer is balanced by a counter mass attached to a lever. The whole unit is hermetically sealed against the atmosphere by die-shaped stainless steel bellows. The details of the machine are given in *Table 1*. The unit is driven by pressure waves from the compressor which is destined for a long life and negligible gas contamination. *Figure 2* gives details of this machine.

The widths of the axial slots cut on the displacer, are kept at 0.2 mm for the 1st stage and 0.15 mm for the second stage. However, it should be noted that the process of cutting these slots is rather cumbersome and a number of cutters have to be sacrificed for this purpose.

After the displacer is machined, it is cleaned and washed in petrol. As the slots on the displacer are very small, there is a possibility of clogging and this can deteriorate the performance of the regenerators. The displacer is placed in a plastic jacket which is then wrapped with multilayer insulation. The whole cold-stage unit is then placed in a vacuum jacket.

Experimental setup

A similar experimental set up, used earlier for the three-stage machine, is installed to evaluate the performance of the machine and also to compare the actual results with the model predictions. For measurements of temperature, PT 100 sensors are installed on the outer part of the casing at the 1st and 2nd stages of expansion. To measure the refrigeration effect at the 2nd stage expansion space, a Manganin heater wire is wound around the casing. A pressure gauge is used to note the maximum and the minimum pressures in the system. To record the pressure and the displacer position variations during a cycle, a pressure sensor and an LVDT are installed. The pressure sensor is installed just at the point where gas enters the displacer unit, while the LVDT is installed on the lever-rod of the cold stage unit, on which the counterweight is mounted. The output signals from both these sensors are fed to an oscilloscope.

Once the system is filled with the gas to a specific charging pressure, the machine is run until the temperatures at the two expansion stages are stabilized. After this, a known power input is applied by the heater wires, changing the temperature distributions of the machine. The new stabilized temperatures are again measured. The power input represents the net refrigeration effect at the 2nd stage for the obtained stable temperature distribution. The pressure and displacer position variations are noted from the oscilloscope.

The power input to the system is measured by the electric power of the motor. Three charging pressures are used to test the performance of the machine: 4, 5 and 6 bar. For all the cases the frequency of the machine is kept at 2.1 Hz. The counterweight position is kept the same for all pressures.

Results and discussion

Cooling behaviour

The minimum temperature obtained from the machine is 45 K with a 6 bar charging pressure and the corresponding 1st stage temperature is 93 K. However, as the charging pressure is reduced the temperature level increases. At 4 bar, the temperatures at the 1st and 2nd stages are 105 and 50 K respectively. It is observed that the change in the 2nd stage temperature with pressure is very small, but the 1st stage temperature is quite sensitive to pressure changes.

Figure 3 gives the cooling curves for 4 and 6 bar charging pressures for comparison purposes. It can be seen that the slope of the cooling curve increases as the charging pressure increases. When the charging pressure is 4 bar, it takes around $4\frac{3}{4}$ h to reach 50 K; while it takes around 3 h 24 min to reach 50 K in the case of 6 bar charging pressure. The decrease in time is essentially due to an increase in the refrigeration effect at higher pressures.

Pressure–displacement curves

Pressure–displacement diagrams are generated for different pressures and temperatures. *Figure 4* gives the pressure–

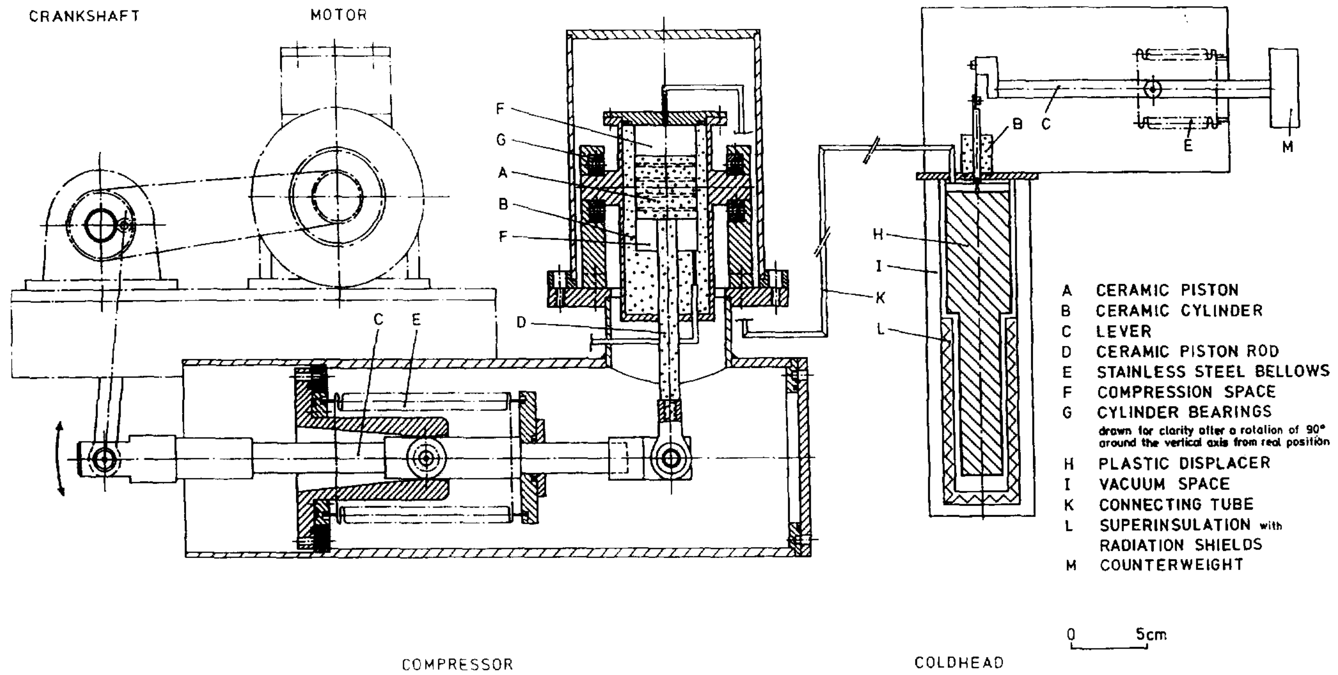

Figure 2 Two-stage split Stirling cryocooler

Table 1 Specifications of two stage Stirling cryocooler (speed = 2.1 rps, connecting tube length = 1000 mm, inner diameter of the tube = 2 mm)

	Compressor space	1st expansion space	2nd expansion space
Diameter (mm)	40	30	21.6
Stroke (mm)	40	8.0	8.0
Number of regenerator slots	—	70	55
Height (mm)	—	87	98
Length (mm)	—	5	3
Width (mm)	—	0.20	0.15

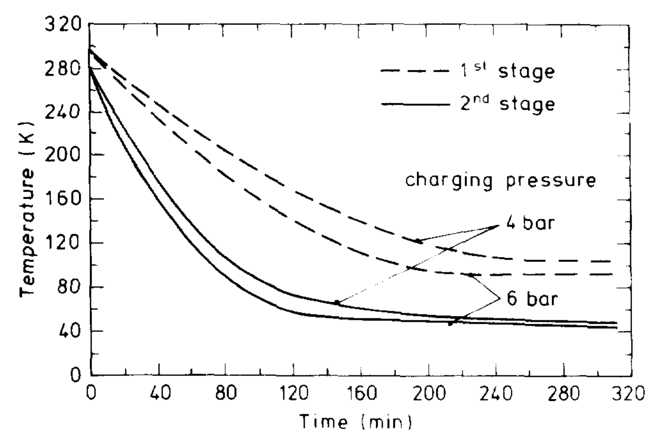

Figure 3 Cooling curves for two-stage Stirling cryocooler

displacement curves for a charging pressure of 5 bar at different temperatures. The curves (a), (b) and (c) show these variations at the 2nd stage temperatures of 250 K, 100 K and 50 K respectively.

It can be seen from these figures that with decreasing temperature the amplitude of the pressure wave also reduces. The maximum and the minimum pressure at 250 K are 9.0 and 2.8 bar respectively, and they are 7.4 and 3.2 bar at 50 K. Also, the phase angle between the pressure and displacer motion increases with decrease in temperature. This increase is found to be of the order of 50% at 50 K, compared to the phase shift at 250 K. The decrease is due to the fact that the viscosity of the working fluid decreases with temperature.

It is noted from these curves that the motion of the displacer is faster at higher temperatures and is sluggish at lower temperatures. This can also be explained by the fact that the viscosity of the working fluid decreases with temperature, and therefore the pressure drop in the system across the displacer is less at lower temperatures. Owing to the smaller pressure drop across the displacer at lower temperatures, its movement is sluggish compared to higher temperatures. This also leads to a reduction of the time for which the displacer position is stationary.

Figure 5 gives the pressure–displacement diagrams for charging pressures 4, 5 and 6 bar at the 2nd stage temperature of 50 K. A similar trend can be noted here regarding displacer motion. The displacer motion is faster at 6 bar compared to the one at 4 bar. This is due to the fact that the pressure drop in the system increases with increasing average pressure.

Comparison of model predictions with experimental results

The developed model is run for all these cases, taking the temperature data from the machine. It is found that the model predictions are a very good match with the experimental results. The comparison of the results is done in terms of pressure ratios, power input to the compressor and the refrigeration effect. The specified compressor power input is at the output of the crankshaft, and 70% transmission efficiency is assumed from the crankshaft to the piston unit. The displacer motion used in the model is based on pressure–displacement curves obtained from the three-stage Stirling machine. It has been found that the pressure–

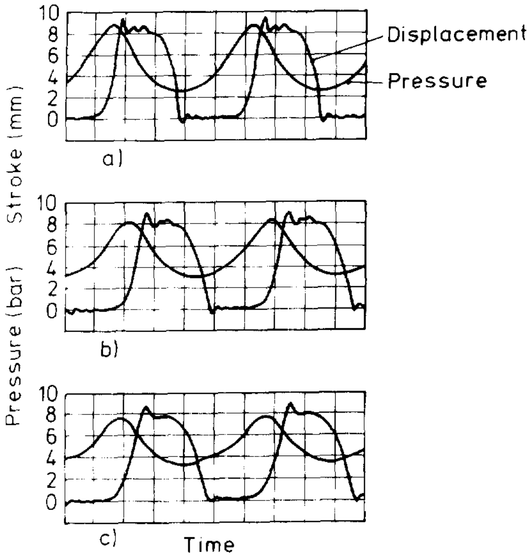

Figure 4 Pressure-displacement diagram for different 2nd stage temperatures: (a) 250 K, (b) 100 K, (c) 50 K

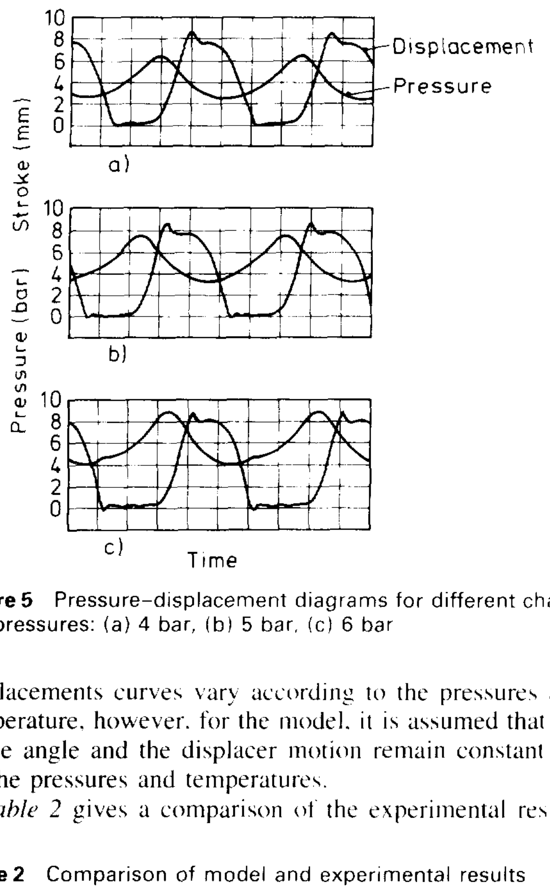

Figure 5 Pressure-displacement diagrams for different charging pressures: (a) 4 bar, (b) 5 bar, (c) 6 bar

displacements curves vary according to the pressures and temperature, however, for the model, it is assumed that the phase angle and the displacer motion remain constant for all the pressures and temperatures.

Table 2 gives a comparison of the experimental results with the predictions of the model. In the first case of 4 bar charging pressure, the lowest temperatures reached are 105 and 50 K; for 5 bar they are 94 and 47 K; while for 6 bar they are 93 and 45 K, for the 1st and 2nd stages, respectively.

In the first case, the prediction of the model in terms of power input and refrigeration effect is very good. The refrigeration effect at 90 K is around 800 mW in this case, while that predicted by the model is 850 mW. The actual and predicted pressure ratios at these temperatures are 2.5 and 2.2, respectively, and the power input is 9.9 W and 8.7 W, respectively. Also, in case 2, for a charging pressure of 5 bar, a very good match is found. In the third case of 6 bar charging pressure, the model predicts a refrigeration effect of 1.3 W at 85 K, while, in effect, it is 1 W. However, the pressure ratios and power input show a good match.

Overall, the model predictions are quite close to the actual values. A discrepancy of 5–30% is observed between the predicted and measured refrigeration effects and is of around 5–15% between predicted and measured compressor power inputs at the crankshaft. There could be many reasons for these deviations. Firstly, the loss calculations are based on the analytical expressions. There could be some parameters, like heat transfer coefficient, friction factors or thermophysical properties, which can vary by a significant amount in the actual experiments without being taken into account. Also, there could be some other losses

Table 2 Comparison of model and experimental results

		Temperature (K)		Parameters					
				P_{max}/P_{min} (bar)		Power input (W)		Refrigeration effect (W)	
Sr. number	Charge pressure (bar)	Stable temperature (with no heat load)	Stable temperature (with heat load)	Model results	Experimental results	Model results	Experimental results	Model results	Experimental results
1	4	I: 105 II: 50	145 90	2.21	2.5	8.7	9.9	0.85	0.80
2	5	I: 95 II: 47	139 94	2.2	2.4	10.5	11.35	1.13	1.0
3	6	I: 93 II: 45	130 85	2.1	2.2	14.2	16.5	1.3	1.0

Table 3 Performance evaluation for the different strokes

Sr. number	Stroke (mm)	Stable temperatures with no heat load (K) Stage I	Stage II	Stable temperatures with heat load of 1 W at 2nd stage (K) Stage I	Stage II
1	8	93	45	130	85
2	7	100	49	138	95
3	6	115	55	145	104
4	5	121	61	154	115
5	4	139	68	175	141

which could not be taken into account owing to the limited knowledge of the processes.

Despite such discrepancies, the importance of the computer modelling is beyond doubt. It gives valuable information regarding the machine performance before the actual machine is fabricated. The parametric study can be very important with regard to design optimization and machine performance.

Test of optimisation

The two-stage machine has been considered as an optimized machine based on the optimization studies carried out with the help of the model. The design parameters optimized are the diameters of regenerator and expansion space, $D(1)$ and $D(2)$, and the stroke of the displacer, S_D. In order to ascertain that this machine gives the best performance compared to any other configuration in terms of diameters, a comparative study should be carried out. This calls for different displacers of various diameters, involving great efforts in terms of time and workload, making the exercise practically unfeasible. However, the present machine can be operated at different strokes, for the existing diameter configuration, keeping the maximum stroke as 8 mm. This is possible by introducing discs of different thickness, in the space above the displacer, limiting the stroke of the displacer.

A comparative study was carried out in this regard and the performance of the machine was evaluated for the displacer stroke ranging from 8 mm to 4 mm in the interval of 1 mm. *Figure 6* gives this comparison. The solid line in this figure is the least square fit through the experimental data. It is observed that as the stroke of the displacer increases from 4 mm to 8 mm, the end temperature, T_{MIN}, for the 2nd stage decreases from 68 K to 45 K. T_{MIN} for the 1st stage, in this case, decreases from 139 to 93 K. However, it should be noted from this curve that as the stroke of the displacer increases from 4 mm to 8 mm, the slope of the curve dT_{MIN}/dS_D decreases, indicating that the extreme value of the curve is likely to lie at a somewhat higher value than 8 mm. *Figure 7* gives the change in absolute value of the slope of this curve with the stroke. The value of the slope can be predicted to be zero at a slightly higher value than 8 mm, indicating that the lowest T_{MIN} value could be obtained at this stroke. However, higher values of stroke may be difficult to realise in practice for a free displacer machine when the average pressure of the working fluid is 6 bar.

The present study, therefore, shows that the performance of the present machine is close to its optimum, in regard to the T_{MIN} value, when the stroke of the machine is 8 mm for the present diameter configuration.

Conclusion

A two-stage, free-displacer Stirling cryocooler is developed. The machine is based on model simulations and optimization of the design which were carried out previously. The refrigeration capacity of the machine is found to be around 1 W at 85 K at 6 bar charging pressure, while the lowest no-load temperature achieved is 45 K. A detailed experimental investigation was carried out in order to evaluate the performance of the machine under different working conditions. Different aspects, like pressure variations and displacer motion, cooling curves, refrigeration effect at different pressures, etc., were studied. The predictions of the model were found to be close to the actual results obtained from the machine. It was also experimentally found that the present stroke value is very near to its optimum, with regard to the T_{MIN} value, for the present diameter configuration. However, one should note that this machine is not a fully optimized one. The lengths of both the regenerators, average pressure in the system, phase angle between pressure and displacement variations, etc., have assumed values. In order to evaluate a fully optimum configuration, all these parameters should be taken into account, which calls for a multi-parameters optimization routine.

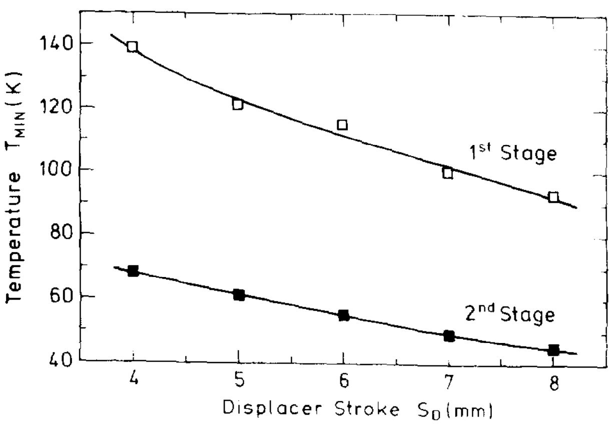

Figure 6 Effect of displacer stroke on minimum temperature

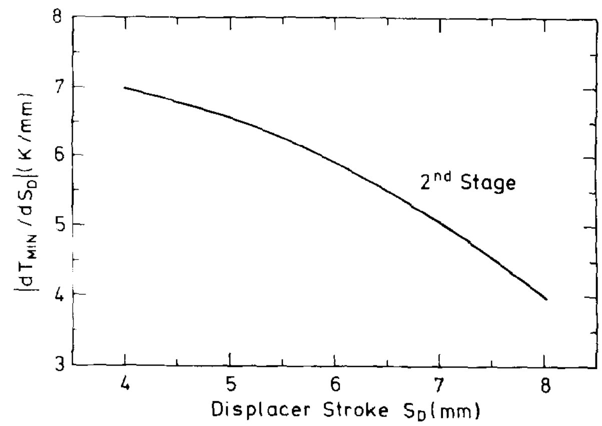

Figure 7 Variation of (dT_{MIN}/dS_D) with stroke

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